STATISTICAL MODEL OF QUANTIZED DCT COEFFICIENTS

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Abstract: In this paper we propose a two-parameter probability model of quantized DCT coefficients for image and video (i.e. JPEG&MPEG format materials), which is based on empirical observation of their statistics in the quantized DCT coefficient domain. The parameters are evaluated in terms of the maximum likelihood criterion which is applied to different frequency coefficients space. The experimental results show that quantized DCT coefficients' distribution modeled by Generalized Laplacian performs better than by Laplacian.

Key words: Digital Watermarking, Generalized Laplacian Distribution, Maximum likelihood Estimation

1. Introduction

Due to the earlier role it plays in still images and video compression algorithms, non-quantized AC DCT coefficients distribution has been thoroughly studied, probability model based on the DWT domain also emerged[1,2,7]. In particular, several approaches have been proposed to model AC coefficients through analytically probability density functions (pdf)[3,4,5,6,8]. With the increasing influence in digital watermarking and steganography, the probability density model are investigated over again, Barni,et al [4] proposed a statistical modeling of full frame DCT coefficients. Although the distribution model above bring conveniency in DCT-block and/or full frame DCT for watermarking and other application, once the material is JPEG/MPEG-2, the distribution models above will be not appropriate. In order to develop a robust and efficient watermarking for JPEG/MPEG-2 media, the distribution of nonzero AC coefficients (hereafter called compression domain) is here investigated. Based on previous study on distribution of block-DCT and full frame DCT, the coefficients are first assumed to follow a Generalized Laplacian distribution. The parameters are then evaluated in terms of maximum likelihood(**ML**) criterion which is applied to different frequency coefficients space.

The rest of the paper is organized as follows. In Section 2, a statistical model is proposed. In Section 3, a parametric model is applied to solve two parameters in each coefficient space. In Section 4, the model's fitness is verified through comparative experiments. In section 5, the conclusion is given.

2. Statistical model for compression coefficients

In many watermarking and steganography methods, the DCT coefficients are usually modified to encode our hidden information. Because the DC coefficients' (which represent the mean value within a block) modifications are more likely to result in perceptible blocking artifacts, only the nonzero AC coefficients are utilized during the encoding. Zero valued coefficients are also skipped because they always occur in featureless areas of an image or video where modifications are most likely create visible artifacts. So the statistical model is discussed in the remaining AC coefficients.

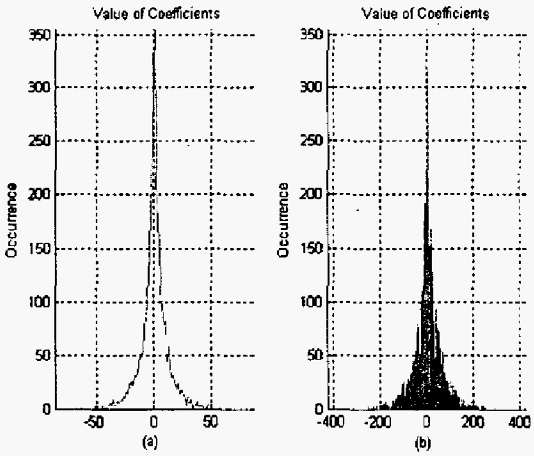

Figure 1.(a)histogram of compression coef(1,2) (b)non-compression coefficients(1,2)(Goldhill)

Example histograms for compression domain and non-compression domain are respectively plotted in Figure 1(a) and Figure 1(b). Compared with Figure 1(b), Figure 1(a) curve is shown to be more sharply peaked at zero, with more extensive tails. The intuitive explanation is that compression domain has more zero values than non-compression domain (which is after DCT transform and before quantization).

This notion is evident statistically that the coefficients in compression domain of images or video have significantly non-Gaussian probability density functions (pdf's). Specifically the coefficient of kurtosis k (fourth moment divided by squared variance) is typically well above the value that one expects for a Gaussian pdf.

Based on analysis above, we propose a two-parameter generalized Laplacian distribution, also used by Simoncelli [2] et al in DWT domain:

$$P_{s,p}(x) = e^{-|x/s|^{p}} \tag{1}$$

the distribution is zero-mean and symmetric, and the parameters{s, p} are directly related to the second and fourth moments. Specifically (after consultation with an integral table) we obtains:

$$\sigma^2 = \frac{s^2\Gamma\left(\frac{3}{p}\right)}{\Gamma\left(\frac{1}{p}\right)}, \quad \kappa = \frac{\Gamma\left(\frac{1}{p}\right)\Gamma\left(\frac{5}{p}\right)}{\Gamma^2\left(\frac{3}{p}\right)} \tag{2}$$

where $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$, the well known "gamma" function. The parameters**{s, p}** are directly related to the second and fourth moments. Specifically where σ^2 is the distribution variance, and κ is the kurtosis.

Given the sample variance and kurtosis of a histogram, we can solve for the two parameters of our model's pdf. As it isn't accurate and highly simple, we provide a precise method to estimate parameters in Section 3.

3. ML estimate of parameter s, p

Let ***coef(i,j)*** denotes a coefficient set of each 8×8 block which vertical and horizontal frequency are respectively i and j($0 \le i < 8, 0 \le j < 8$), $max_{coef(i,j)}$ represents the maximum-absolute ceiling value of ***coef(i,j)***. We construct a set of sub-interval $[x_i, x_{i+1}]$ ($x_1 = -max_{coef(i,j)}$; $x_{length} = max_{coef(i,j)}$; $x_{i+1} - x_i = 1$; ***length*** denotes number of subinterval).

For each ***coef(i,j)***,we solve(numerically) for the parameters***{s,p}*** by minimizing the relative entropy(i.e.,the Kullback-leibler divergence) between a discretized model distribution and the length-bin coefficient histogram:

$$\Delta H(s,p) = -\sum_{i=1}^{length} h_i \log_2 \frac{\int_{x_i-0.5}^{x_i+0.5} p_{s,p}(x)}{h_i} \tag{3}$$

where h_i is the normalized histogram count(frequency) for the ***ith*** histogram bin. The measure $\Delta H(s,p)$ corresponds to the cost (in bits) of encoding the data with an entropy coder that assumes the distribution $p_{s,p}(x)$.

To obtain goodness fit between statistical model and histogram, we utilize maximum likelihood methodology, maximum likelihood is:

$$p^*, s^* = \arg\max_{s,p}\{-\Delta H(s,p)\} \tag{4}$$

where p^*, s^* is estimated value of **p,s** through maximum likelihood criterion.

4. Experimental results

To demonstrate the statistical model is good fit, we perform two class experiments of which one is model curves' fitness to practical data histogram, the other is relative entropy value comparison between Laplacian and Generalized Laplacian. The Laplacian model whose pdf is set as $p_\lambda(x) = \frac{\lambda}{2}e^{-\lambda|x|}$ is chosen as frame of reference, for it has also been used to describe coefficient histograms that are peaked at zero[8].

We performed a great deal of experiments on various images and video frames in order to evaluate the accurate characteristic of our statistical model. In this section, we present two of the most significant results relating to the intensity Goldhill and Lena image (i.e. JPEG format), which are the case at coef(1,2) of image Goldhill and Lenna .First, from Figure 2 and Figure 3, it could be shown clearly that curve of the Generalized Laplacian model is more close to histogram than the curve of Laplacian model. Table 1 presents the value of parameters and relative entropy of two model, relative entropy reflect the extent of fit between histogram and statistical model, the Generalized Laplacian model has a smaller ΔH than the Laplacian model for each coefficient, which means the prior is a closer fit model.

5.Conclusion

In this paper the statistical model of DCT compression domain has been proposed. According to previous analyses on histogram shape of nonzero AC coefficient, the probability density function of compression domain has first been assumed to follow Generalized Laplacian distribution. The experimental results above show that quantized DCT coefficients' distribution modeled by Generalized Laplacian performs better than by Laplacian.

Table 1. The value of parameters and ΔH correspond to different non-zero AC coefficients (Golhill&Lena)

	Goldhill					Lena				
	Laplacian		Generalized Laplaican			Laplacian		Generalized Laplaican		
	λ	ΔH	s	p	ΔH	λ	ΔH	s	p	ΔH
C01	0.10769	49858	2.154	0.39452	18016	0.028055	10677	1.9434	0.43245	4580.5
C02	0.11956	49383	2.8992	0.10124	19677	0.055406	10619	2.1716	0.51931	4518.9
C03	0.27075	49257	1.9143	0.74436	17203	0.19981	10537	2.83	0.21995	1293.4
C04	0.27013	49301	1.476	0.18559	16347	0.067813	10682	2.83	0.77198	1287.1
C05	0.19616	49730	1.476	0.18559	17154	0.060688	10590	1.7256	0.22086	3133.1
C06	0.45694	49670	1.1254	0.2619	13493	0.16088	10490	1.7483	0.35231	2664.4
C07	0.52581	49212	0.81281	0.23645	13288	0.15288	10533	1.7256	0.23484	1767.1
C08	0.36431	49288	2.1368	0.43077	15578	0.19731	10529	2.83	0.21995	1251.5
C09	0.404	49221	1.1397	0.81967	15275	0.3585	10382	1.3546	0.20343	3277.3
C10	0.67281	49173	1.4451	0.3666	12177	0.68119	10353	1.9857	0.51708	857.98
C11	0.69106	49227	1.41292	0.5950	11880	0.31731	10465	1.074	0.26341	2889.6
C12	0.62975	49247	1.36002	0.67267	12465	0.22531	10493	3.2039	0.49411	227.87
C13	1.0134	49211	1.3157	0.61551	9936.9	0.30844	10458	1.4236	0.86309	2779.4
C15	1.0314	49521	1.44187	0.44758	9461.8	0.41894	10399	3.2752	0.56143	3225.2

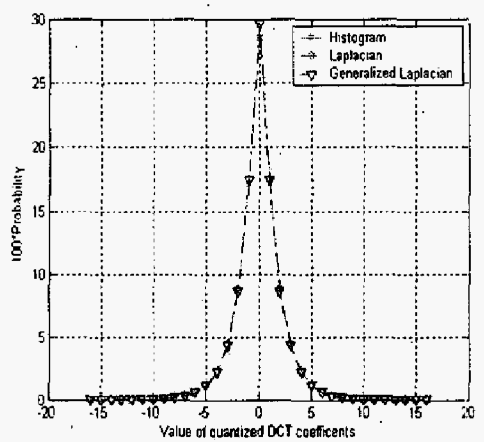

Figure 2. Laplacian&Generalized Laplacian fit to histogram of coef(1,2)(Goldhill)

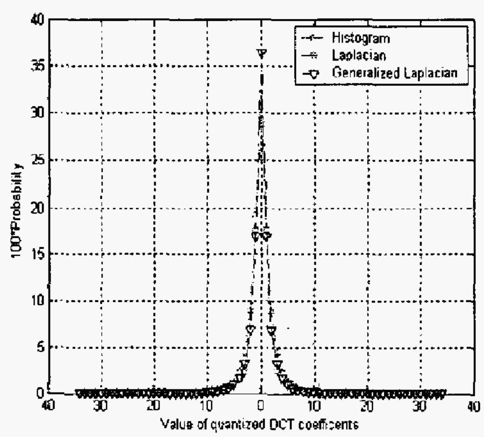

Figure 3. Laplacian&Generalized Laplacian fit to histogram of coef(1,2)(Lenna)

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