On Role Mappings for RBAC-based Secure Interoperation

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Abstract—The inter-domain role mapping is a basic method for facilitating interoperation in RBAC-based collaborating environments, where each domain employs Role Based Access Control (RBAC) to specify access control policies. Prior to concrete interoperation, one important problem is to establish role mappings. Two issues are involved in the establishing process. The first one is to generate role mappings while respecting RBAC states such as separation of duty (SoD) constraints. On the other hand, administrative works of RBAC policies are sometimes needed to generate mappings. This paper investigates these two problems, mostly from the computational perspective. In particular, we study how to find a set of roles appropriate for mappings and how to fulfill interoperation requests; it turns out that most of corresponding problems are NP-complete. Further, several useful subcases of these problems are identified. We also motivate and support partial interoperation by imposing constraints on interoperation requests. When administrative works are necessary, we examine how to minimize administrative cost; the result is that one subcase of the problem reduces to the “minimal set cover” (MSC) problem. We demonstrate that approaches to MSC can be applied to this problem, even though they are not totally equivalent. Finally, a discussion on how administrative operations made to RBAC states may influence interoperability is presented as well.

Keywords—role mappings, secure interoperation, RBAC, administration

I. INTRODUCTION

In a collaborating environment, several domains may interoperate with each other to complete a task. A domain is a member of the collaborating environment, which manages some resources and enforce its own access control policies. Specifically, a user wanting to access resources controlled by another domain B. Not surprisingly, each domain wants to control such cross-domain accesses. A tempting approach is to let B define a set of permissions available for u so that u can only perform permitted operations. Though intuitive and easy to implement, it does not fit in real cases in two aspects. Firstly, B has no prior knowledge of u, thus cannot authenticate u or make authorization decisions for u. Second, there could be many foreign users like u; it seems too much burden on B to specify and enforce policies for each foreign user.

Since Role-based access control (RBAC) [8], [20] has several advantages such as being policy-neutral and being able to support a wide range of access control requirements, it is widely supported in commodity operating systems and database systems. Therefore, RBAC is deployed in many organizations to model and enforce their access control needs. In an RBAC-based collaborating environment, domains’ access control policies are specified using RBAC models. Due to the importance of RBAC, we focus on the secure interoperation in such contexts.

Mapping roles between A and B appears to be a promising approach to secure interoperation in such an RBAC-based collaborating environment [7], [10], [11], [14], [21], [24], [25]. Most of these works focused on either the discovery of role mappings or applications built on top of role mappings; they paid little attention to the problem that which set of roles should be mapped (i.e., the role mapping generation). Before the interoperation, A issues a request to establish role mappings between A and B. Usually, the request is a tuple \((Key_A,r_A,PS,Key_B)\), where \(Key_A\) is a signature of A by which B can verify the requesting domain, \(r_A\) is a role in domain A, \(PS\) is a set of permissions of B which members of \(r_A\) wish to execute to accomplish a task, and \(Key_B\) is a signature of B identifying the receiver of the request. A role mapping defines inter-domain inheritance semantics between A’s role hierarchies and those of B. Assuming that B maps its role \(r_B\) to \(r_A\), \(r_A\) obtains all the permissions available to \(r_B\). In other words, users of \(r_A\) are assigned those permissions. Then, \(u\) can authenticate herself or himself and act as \(r_B\) in B by a credential issued by A showing that \(u\) is a member of \(r_A\).

However, prior to concrete interoperation, several challenges may arise for B to deal with such requests. First of all, \textit{Separation of Duty} (SoD) is widely accepted as a fundamental principle in computer security and is supposed not to be violated [2], [4], [1], [15]. In RBAC contexts, \textit{Static Mutually Exclusive Roles} (SMER) constraints are employed to enforce SoD [3], [15]; thus SoD constraints are considered as violated if SMER constraints are not observed. But, it may cause a violation of B’s SMER constraints that assignment all permissions in \(PS\) to \(r_A\) by mapping B’s roles. For example, suppose that \(r_A\) tries to acquire both \(p_1\) and \(p_2\) (i.e., \(\{p_1,p_2\} = PS\)), and that B imposes an SoD constraint that no users be assigned both \(p_1\) and \(p_2\). Some SMER constraints should be in place to enforce this SoD. Then it would be a potential violation if \(\{p_1,p_2\}\) are granted through role mappings, because any user of \(r_A\) is assigned the two permissions.

Secondly, good chance is that B’s access control policies are specified beforehand; to either fully or partially satisfy requests, B may be willing to adjust B’s access control policies [21]. In particular, since there do not always exist suitable roles for mappings, administrators may change RBAC policies. They may resort to creating new roles, adjusting permission assignments, and altering role hierarchies. Hence, natural problems are how to choose adjustment strategies when multiply alternatives are available, and which set of administrators to implement needed administrative operations for a particular request. Finally, when no or defective
changes are made to B’s policies, it may happen that B fails to fulfill the request; that is, only a proper subset of the requested permissions needed to accomplish a task is shared. In this case, we may settle if some subtask could be done by the interoperation.

In this paper, we study above-mentioned problems during the generation of inter-domain role mappings. The first is to obtain maximum interoperability while preserving the integrity of SMER constraints (thus SoD constraints). Additionally, we propose to put constraints on requested permissions to support partial interoperation. We attempt to determine whether B can fulfill an interoperation request, and, if not, whether it is possible for B to achieve it by adjusting its policies. In the latter case, we study how to find a minimal set of administrators when administrative operations are due. All these three problems are proved NP-complete. We give an integer programming approach to searching the minimal set of administrators.

The rest of the paper is organized as follows. Preliminary definitions are presented in Section II. In Section III and IV, we investigate the problem of generating role mapping instances. In Section V, we study the notion of compatibility and show the necessity of making changes to RBAC policies. Section VI gives the related work on secure interoperation in collaborating environments. Finally, we conclude in Section VII.

II. PRELIMINARIES

A. Role-based access control

The RBAC model used in this paper consists of seven components: $U$ (a set of users), $P$ (a set of permissions), $R$ (a set of roles), $UA$ ($UA \subseteq U \times R$ assigns roles to users), $PA$ ($PA \subseteq R \times P$ assigns permissions to roles), $RH$ ($RH \subseteq R \times R$ enables permission inheritance between roles), and $C$ (a set of Static Mutually Exclusive Roles (SMER) constraints stating what user-role assignments are not allowed).

Let the reflexive and transitive closure of role $R$ be $RH^r$. We denote $(r_i, r_j) \in RH^r$ by $r_i \succ r_j$ and say that $r_i$ is senior to $r_j$ in the sense that $r_i$ inherits all permissions of $r_j$. We borrow the expressions of SMER constraints in [3], [15]. A $t$--$m$ SMER constraint $c = \text{smcr}\{\{r_1, r_2, \cdots, r_m\}, t\}$ forbids $t$ more roles in $\{r_1, r_2, \cdots, r_m\}$ from being assigned to a single user.

Definition 1 (RBAC state): An RBAC state $\phi$ is a 4-tuple $(R, PA, RH, C)$.

Unless otherwise stated, we assume B’s RBAC policies are in state $\phi = (R, PA, RH, C)$. Given $r \in R$, $R' \subseteq R$, $p \in P$, and $P' \subseteq P$ in a state $\phi$, we define $\text{PermSet}_\phi(r)$, $\text{PermSet}_\phi(R')$, $\text{RoleSet}_\phi(p)$, and $\text{RoleSet}_\phi(P')$ as follows:

$$\text{PermSet}_\phi(r) = \{p \in P | \exists r_i \in R[r_i \succ r \land (r_1, p) \in PA]\}$$

$$\text{PermSet}_\phi(R') = \bigcup_{r \in R} \text{PermSet}_\phi(r)$$

$$\text{RoleSet}_\phi(p) = \{r \in R | (r, p) \in PA\}$$

$$\text{RoleSet}_\phi(P') = \bigcup_{p \in P'} \text{RoleSet}_\phi(p)$$

Note that $\text{PermSet}_\phi(r)$ includes all (i.e. both explicitly assigned and inherited) permissions of $r$, whereas $\text{RoleSet}_\phi(p)$ only contains roles which are explicitly assigned $p$.

B. Administration of RBAC

Several administration models of RBAC have been proposed in literature [5], [6], [19]. In this paper, we employ the administrative model SARBARC [6] to specify the administration of RBAC policies. The administration involves controlling updates to the role hierarchies $RH^r$ and the relations $UA$ and $PA$. The following definitions about administration are due to Crampton et al. [6].

Definition 2: (Administration of RBAC) The administration of RBAC policies is controlled by a binary relation $\text{admin-authority} \subseteq R \times R$, which specifies the administrative roles and their authorities.

We say a role $ar \in R$ is an administrator if there exists $(ar, r) \in \text{admin-authority}$. The set of roles that $ar$ controls is denoted as $C(ar) = \{r \in R | (ar, r) \in \text{admin-authority}\}$. The duty of an administrator is represented by the administrative scope. For $r \in R$ and $R' \subseteq R$, let $\uparrow r = \{r' \in R | r' \succ r\}$, $\downarrow r = \{r' \in R | r \succ r'\}$, $\downarrow R' = \bigcup_{r \in R'} \uparrow r$ and $\uparrow R' = \bigcup_{r \in R'} \downarrow r$.

Definition 3 (Administrative Scope): The administrative scope of an administrator $ar$, denoted as $S(ar)$, is defined to be:

$$S(ar) = \{ r \in C(ar) | \uparrow r \cap C(ar) \subseteq C(ar) \}$$

The strict administrative scope of $ar$ is defined to be $S^+(ar) = S(ar) \setminus C(ar)$. For the administrator set $AR \subseteq R$, we have $S(AR) = \{ r \in C(AR) | \uparrow r \cap C(AR) \subseteq C(AR) \}$ and $S^+(AR) = S(AR) \setminus C(AR)$, where $C(AR) = \bigcup_{ar \in AR} C(ar)$.

An administrator $ar$ is responsible for controlling roles in $S(ar)$. Briefly, several administrative operations are at $ar$’s disposal such as addEdge and deleteEdge. For more details on administrative scope and operations, we refer the readers to [5], [6].

We will not usually concern ourselves with the test of operations’ conditions and simply assume that operations would succeed. We denote the set of operations that $ar$ can execute by $OP(ar)$ and that a set $AR$ of administrators can perform by $OP(AR)$.

We assume that $AR$ can manipulate a permission $p$ only if at least one of the roles assigned $p$ is under $AR$’s control, i.e., $\text{RoleSet}_\phi(p) \cap S(AR) \neq \emptyset$. The reason for this consideration is that there otherwise might be meaningless roles or hierarchies in RBAC states. We will omit concrete operation commands if they are clear from the context.

RBAC states evolve with the executions of administrative operations.

Definition 4: (RBAC State Transformation) Given a set $AR$ of administrators and an RBAC state $\phi$, we say the RBAC policies transform from $\phi$ to $\phi'$ by a sequence of administrative operations $\sigma_{i_1} \cdot \sigma_{i_2} \cdots \sigma_{i_m}$, where $\sigma_i \in OP(AR)$, $1 \leq i \leq m$. We denote this transformation as $\phi \xrightarrow{AR} \phi'$. We omit the label $AR$ when it is obvious from the context.

Example 5: A running example is shown in Figure 1. Consider an RBAC state $\phi$ shown in Figure 1. According to the definition of administrative scope, we have $S(\{a_2\}) = \{a_2\}$, $S(\{a_3\}) = \{a_3\}$ and $S(\{a_2, a_3\}) = \{a_2, a_3, r_7\}$ for example.

All further examples in this paper are based on the RBAC state $\phi$ in Figure 1, and other states deriving from $\phi$ are elucidated by giving differences between them and $\phi$. 

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III. INTEROPERATION BASED ON INTER-DOMAIN ROLE MAPPINGS

Definition 6 (Interoperation request): An interoperation request is a tuple $T = (\text{Key}_{A}, r_{A}, \mathbb{P}, \text{Key}_{B}, \Sigma)$, where

1) $\text{Key}_{A}$ is a signature of $A$ by which $B$ can verify the requesting domain,
2) $r_{A}$, named the requesting role, is a role in domain $A$,
3) $\mathbb{P}$ is a set of permissions of $B$ which members of $r_{A}$ wish to execute to accomplish a task,
4) $\text{Key}_{B}$ is a signature of $B$ identifying the receiver of the request, and
5) $\Sigma$ is a set of constraints on $\mathbb{P}$.

When elements in $T$ are clear from context we may omit them in $T$.

The syntax of constraints in $\Sigma$ is defined as follows:

$$\sigma \triangleq p | \sigma \land \sigma | \sigma \lor \sigma | \sigma \rightarrow \sigma$$

We now explain the intuition of introducing $\Sigma$ into interoperation requests. Since it is not always the case that all the requested permissions, which is needed for the task $t$, are shared, we may be as well satisfied if part of $t$ (e.g., sub-tasks) can be accomplished. However, only certain subsets of $\mathbb{P}$ provides users with adequate rights. For example, users with the capability of read but no write may fail to work on any subtasks. Since all permissions in $\mathbb{P}$ are desired, it is not necessary to involve negations in constraints. Given a request with constraint $\Sigma$, we implicitly revise $\Sigma$ to be $\Sigma \lor (\Lambda \mathbb{P})$. This is because sharing all permissions in $\mathbb{P}$ is always desired.

Some typical constraints may be $(p_{1} \land p_{2}) \lor (p_{3} \land p_{4})$ and $(p_{3} \land (p_{4} \rightarrow p_{5})) \lor p_{6}$. The first one means that the interoperating task $t$ may be divided into two subtasks $t_{1}$ and $t_{2}$, which requires $p_{1} \land p_{2}$ and $p_{3} \land p_{4}$, respectively. The expression $p_{4} \rightarrow p_{5}$ in the second constraint, which states $p_{5}$ must be granted if $p_{4}$ is so, appears tricky. This expression is intended to enable task integrity; $p_{4}$ and $p_{5}$ are not a must for a subtask, but $p_{4}$, if present, needs to be accompanied by $p_{5}$. For example, if the permission commit is shared, another permission rollback should also be granted; otherwise, users would fail to abandon what they committed before.

Definition 7 ($\sigma$-satisfiability): Given a set $R$ of roles and a constraint $\sigma$ we define a relation $\models$ based on the structure of $\sigma$:

1) $R \models p$ iff $p \in \text{PermSet}_{\sigma} (R)$, 
2) $R \models \sigma_{1} \land \sigma_{2}$ iff $R \models \sigma_{1}$ and $R \models \sigma_{2}$,
3) $R \models \sigma_{1} \lor \sigma_{2}$ iff $R \models \sigma_{1}$ or $R \models \sigma_{2}$, and
4) $R \models \sigma_{1} \rightarrow \sigma_{2}$ iff $R \models \sigma_{2}$ whenever $R \models \sigma_{1}$.

We write $R \models \Sigma$ if $R \models \sigma$ for all $\sigma \in \Sigma$. We say $\sigma$ (respectively, $\Sigma$) is satisfied against $R$ if $R \models \sigma$ (respectively, $R \models \Sigma$).

Upon receiving a request $T$ and verifying the signatures, $B$ creates role mappings by a searching algorithm in role hierarchies based on the subsumption relationships between roles’ permission sets and $\mathbb{P}$. The algorithm also needs to take into account SMER constraints in $B$ and $\Sigma$ in $T$. The algorithm outputs a role mapping instance (defined below) for each request. Given a set $R$ of roles and an SMER constraint $c = \langle \{r_{1}, \cdots, r_{m}\}, t\rangle$, we say $R$ satisfies $c$ if $| R \cup \{r_{1}, \cdots, r_{m}\} | < t$; we say $R$ satisfies $c$ iff for all $c \in C$, $R$ satisfies $c$.

Definition 8 (Role mapping instances): A role mapping instance $RM_{\sigma}(T)$ is a tuple $(r_{A}, Q)$, where $r_{A}$ is the requesting role in $T$, and $Q$ is a set of roles $(r_{1}, r_{2}, \cdots, r_{n})$ where $r_{i} \in R, 1 \leq i \leq n$. $RM_{\sigma}(T)$ should meet the following conditions.

1) $\text{PermSet}_{\sigma}(Q) \subseteq \mathbb{P}$,
2) $Q$ satisfies $C$, and
3) $Q \models \Sigma$.

Usually, we prefer to map the most senior roles while permissions beyond $\mathbb{P}$ are not shared consequently. That is, $Q = \{ r \in R \mid \text{PermSet}_{\sigma}(r) \subseteq \mathbb{P} \land \neg \exists r_{1} \in R \mid r \models r_{1} \land \text{PermSet}_{\sigma}(r_{1}) \subseteq \mathbb{P} \}$.

Example 9: Suppose that $A$ issues a request $T_{1}$ to $B$.

$$T_{1} = \left( \text{Key}_{A}, r_{A}, \{p_{1}^{1}, p_{3}^{2}, p_{5}^{3}, p_{1}^{0}, p_{1}^{1}, p_{1}^{2}\}, \text{Key}_{B}, (p_{1}^{2} \land p_{1}^{2}) \lor (p_{3}^{1} \land p_{3}^{1}) \right)$$

According to the definition of role mapping instances, an instance $RM_{\sigma}(T_{1})_{1} = (r_{A}, \{r_{12}\})$ could be generated for $T_{1}$. Since $\text{PermSet}_{\sigma}(\{r_{7}, r_{8}\}) = \{p_{1}^{1}, p_{3}^{2}, p_{5}^{3}, p_{1}^{0}, p_{1}^{1}, p_{1}^{2}\}$, $RM_{\sigma}(T_{1})_{1}$, if enforced, enables maximum interoperability in terms of $T_{1}$; thereafter, members of $r_{A}$ in domain $A$ are treated as $r_{7}$ and $r_{8}$ in domain $B$ and thus could perform corresponding operations. On the other hand, other role mapping instances are also of value according to $T_{1}$’s constraints $(p_{1}^{2} \land p_{1}^{2}) \lor (p_{3}^{1} \land p_{3}^{1})$. For example, $RM_{\sigma}(T_{1})_{2} = (r_{A}, \{r_{12}\})$ is an alternative because $RM_{\sigma}(T_{1})_{2}$, if enforced, could allow members of $r_{A}$ both $p_{1}^{2}$ and $p_{1}^{2}$. Another instance $RM_{\sigma}(T_{1})_{3} = (r_{A}, \{r_{7}\})$ is also legal. However, $RM_{\sigma}(T_{1})_{2}$ and $RM_{\sigma}(T_{1})_{3}$ do not provide full interoperability demanded by $T_{1}$. The reason why $RM_{\sigma}(T_{1})_{1}, RM_{\sigma}(T_{1})_{2}$, and $RM_{\sigma}(T_{1})_{3}$ co-exist is that computing and supporting instances like $RM_{\sigma}(T_{1})_{1}$ is difficult, whereas generating $RM_{\sigma}(T_{1})_{2}$ and $RM_{\sigma}(T_{1})_{3}$ are relatively easier. Hence, one can choose among them depending on the requirements.

IV. VARIABLE INTEROPERABILITY

Owing to the flexibility for the definitions of $PA$ and $RH$ in RBAC states, RBAC is able to capture diversified access control requirements. This feature also leads to a set of feasible role mapping instances for each request. In this section, we concentrate on how to choose appropriate instances.
A. Maximal Interoperability

For a collaborating task $t$, suppose that $A$ issues a request $T = (KeyA, r_A, PS, KeyB, \Sigma)$. Both $A$ and $B$ can demand “all or nothing” in terms of interoperability by modifying $\Sigma$ into $\Sigma \rightarrow (\land PS)$. Correspondingly, members of $r_A$ in $A$ can perform either all operations embodied in $PS$ or nothing.

In this setting, role mapping instances $(r_A, Q)$ to have to fulfill two conditions: (1) $PermSet_0(Q) = PS$ and (2) $Q$ satisfies $C$. We refer to these instances as maximal instances and the problem of generating maximal instances as $\maximal_0(T)$. Note that the problem $\maximal_0(T)$ arises only when it is assured that there exists a role mapping instance with maximal interoperability, which is covered in Section V. $\maximal_0(T)$ is of high computational complexity. As shown in Theorem 14, even determining the existence of maximal instances is NP-complete.

We now examine some useful subcases of $\maximal_0(T)$. First of all, we make some assumptions. Assume $B$’s state $\phi = (R, PA, RH, C)$. Let $R_m = \{r \in R \mid PermSet_0(r) \subseteq PS\}$. We view $\maximal_0(T)$ as finding a subset $R'$ of $R_m$ such that (1) $PermSet_0(R') = PS$ and (2) $R'$ satisfies $C$. If $R'$ exists, then let $Q = R'$ and $\maximal_0(T)$ is addressed.

1) An Intractable subcase of $\maximal_0(T)$: SMER constraints, as an enforcement mechanism for SoD constraints, are an unique component in the sense that they describe forbidden authorization scenarios, while other components of RBAC states what are allowed. Not surprisingly, SMER constraints would have important impacts on $\maximal_0(T)$. To explore this aspect, we make some simplifications. We refer to as $\maximal_0(T)$ the subcase of $\maximal_0(T)$ where the following conditions are satisfied:

1) $RH = \emptyset$, and
2) $\forall c \in Q \ [c$ is a 2-2 SMER constraint$].$

We give some explanations. The first condition rules out any role hierarchy in $B$‘s RBAC state. Role hierarchies capture the relationships between roles in business and thus mitigate the burden of access control. This seems too restrictive a condition. However, Sandhu et al. excluded role hierarchies from the base model in the RBAC96 family models [20]. Furthermore, recent role engineering techniques also ignored role hierarchies at present [26], [27]; that is, there are no hierarchies between roles. These two situations imply, to some extent, that $RH = \emptyset$ is not too restrictive to be reasonable. The second condition says that only 2-2 SMER constraints exist. These constraints are the most widely accepted type of constraints in literature; basically, they prevent any user from being a member of both roles. However, even $\maximal_0(T)$ is intractable.

Theorem 10: $\maximal_0(T)$ reduces to SAT.

Proof: Given $R_m$ and a maximal instance $RM_0(T) = (r_A, Q)$, for each $r_i \in R_m$, let $x_{r_i} = 1$ if $r_i \in Q$ and $x_{r_i} = 0$ otherwise. For each permission $p \in PS$, if $p \in PermSet_0(Q)$, then there exists $r_i \in R_m$ such that $r_i \in Q$ and $p \in PermSet_0(r_i)$. Assuming $R_m(p) = \{r_i \in R_m \mid p \in PermSet_0(r_i)\}$, we have $p \in PS \iff \bigvee_{r_i \in R_m(p)} x_i$. As a result, we obtain a formula $fol_1 = \bigwedge_{p \in PS} \bigvee_{r_i \in R_m(p)} x_i$.

For each 2-2 constraint $c = \{r_1, r_2, 2\}$, $c$ is satisfied if and only if $\neg(x_1 \land x_2)$ holds, which corresponds to the set $(\neg x_1 \lor \neg x_2) \land (\neg x_1 \lor \neg x_2)$. Denote $fol_2 = \bigwedge_{c \in C} \neg(x_1 \land x_2)$. Let $fol = fol_1 \land fol_2$. Then, a truth assignment of $x_i$ makes $fol$ true if and only if $RM_0(T) = (r_A, Q)$ where $Q = \{r_i \mid x_i = 1\}$ is a maximal instance.

Corollary 11: $\maximal_0(T)$ reduces to SAT.

Proof: Immediately proved from Theorem 10.

2) Two tractable subcases of $\maximal_0(T)$: Given an SMER constraint $c = \{(r_1, \cdots, r_m)\}, t$, we denote the set $\{r_1, \cdots, r_m\}$ as $c.R$. In some cases, the requested permissions $PS$ does not concern the roles that are protected by SMER constraints; we refer to the problem $\maximal_1(T)$ that satisfies (1) as $\maximal_1(T)$. $\forall c \in C [RoleSet_0(PS) \cap \{c.R = \emptyset\}]$. \hspace{1cm} (1)

$\maximal_0(T)$ could happen when $PS$ includes no sensitive operations like “authorizing payment”. A simple testing is sufficient to enable maximal interoperability while reducing the number of roles being mapped: for each $r \in Q$, if $PermSet_0(Q - \{r\}) = PS$, then remove $r$ from $Q$. In addition, Du and Joshi [7] proved that to find a minimal set of roles such that $PermSet_0(R) = PS$ is NP-complete. We can import their greedy algorithms if the minimization is a criterion.

We now relax the restrictions of $\maximal_0(T)$. Since each $t \in \Sigma$ SMER constraint is equivalent to a subset of $t \in \Sigma$ SMER constraints [15], we assume $C$ consists only of $t \in \Sigma$ constraints and denote this $C$ as $C_t$. Besides, this assumption appears reasonable because lots of SoD constraint definitions in terms of roles in literature take the form of $t \in \Sigma$ constraints [1].

For a $t \in \Sigma$ SMER constraint $c = \{(r_1, \cdots, r_i)\}, t$, only when $\{r_1, \cdots, r_i\} \subseteq Q$ would $c_t$ be violated. Thus, it is not hard to collect the set $c_t$ of violated constraints in presence of an instance $(t_A, Q)$. $\forall c_t = \{c \in C_t \mid c.R \subseteq Q\}$. We put the following restrictions on $c_t$ and denote this subcase as $\maximal_1(T)$.\hspace{1cm} $\forall c_1, c_2 \in c_t \ [c_1.R \cap c_2.R \neq \emptyset \rightarrow c_1 = c_2]$, and $\forall p \in PS [\{RoleSet_0(p) = 1\}]$.

$\maximal_0(T)$ simulates the cases where SMER constraints (SoD constraints) are sparse on the set $Q (PermSet_0(Q))$ and a permission is assigned to only one role. In this setting, one can efficiently either generate the maximal instance or report the nonexistence of maximal instances.

1) for each $c_t$, let $test = PS$;
- randomly choose a permission $p$ from $test$
  a) let $c.R = c.R - RoleSet_0(p)$;
  b) let $test = test - PermSet_0(RoleSet_0(p))$;
  c) if $c.R = \emptyset$ then report “no maximal instance” and exit;
2) return the maximal instance $(r_A, RoleSet_0(PS))$.

B. Partial Interoperability

Since maximal interoperability is not guaranteed, we may have to enforce role mapping instances with only partial interoperability. Given a request $T = (r_A, PS, \Sigma)$, we say a role mapping instance $(r_A, Q)$ is partial instance for $T$ if (1) $PermSet_0(Q) \subseteq PS$ and (2) $\Sigma \models Q$. We denote the problem of finding partial instances $\partial_0(T)$.

Now we consider one subcase of $\partial_0(T)$, denoted as $\partial_0(T)_0$ where constraints in $\Sigma$ take the following syntax instead.

\[\sigma \triangleq q \lor q \lor q \lor p \lor q \lor q\]
**Theorem 12:** partialᵦ(T₀) reduces to SAT.

**Proof:** Given a constraint q = q₁ ∨ ... ∨ qₘ, one may randomly choose qᵢ and transforms qᵢ into a formula folᵢ. This is because if qᵢ |= Q then q |= Q. This proof is based on the proof of Theorem 10 and use the denotations there. Since qᵢ takes the form p₁₁ ∧ ... ∧ pₘₙ, we have folᵢ = \( Xᵢ \). Then, a truth assignment of xᵢ makes folᵢ true if and only if if RMᵦ(T₀) = (rᴬ, Q) where Q = \( \{ rᵢ | xᵢ = 1 \} \) is a partial instance.

Though partialᵦ(T₀) is computationally complex in general, we believe that there are useful tractable subcases in practice, especially with the help of SAT solvers. We leave this line as future work.

**V. Compatibility**

Given a role mapping instance RMᵦ(T) = (rᴬ, Q), the potential interoperability between domain A and B is limited to the set PermSetᵦ(Q). This often results in some permissions in \( \mathbb{P} \) being excluded from being shared. For example, assume that there is a request T₂ = \( \{ p₁₀, p₁₁ \} \) in Figure 1. At present, by no means can p₁₁ be granted for cross-domain access through role mappings. However, we wish to share as many permissions (i.e., interoperability) as possible. In this section, we investigate whether it is possible for B to fully satisfy A’s interoperation requests, even when B is willing to adjust its RBAC states.

**Definition 13 (Total Compatibility):** We say that a request T is totally compatible with an RBAC state φ, if there is a role mapping instance RMᵦ(T) = (rᴬ, Q) such that PermSetᵦ(Q) = \( \mathbb{P} \). We denote this as compatibleᵦ(T), and say T is compatible with φ if compatibleᵦ(T) does not hold.

Only when a request is totally compatible in an RBAC state, there stands a chance for all permissions in the request to be shared. However, it is computationally complex to determine this.

**Theorem 14:** Checking compatibleᵦ(T) is NP-complete.

**Proof:** A nondeterministic machine can guess a tuple \((rᴬ, Q)\) and verify the conditions in the role mapping instance definition and whether PermSetᵦ(Q) = \( \mathbb{P} \) in polynomial time. Therefore, checking compatibleᵦ(T) is in NP.

We now show checking compatibleᵦ(T) is NP-hard by reducing a known NP-complete problem, the minimal cover problem, to one of its special cases. The problem states: given a family \( F = \{ F₁, \ldots, Fₙ \} \) of subsets of a finite set \( Y = \{ y₁, \ldots, yₙ \} \) such that every element in Y belongs to at least one member of F, and a positive integer \( K \leq |Y| \), the problem is to check whether there exists an \( F' \subseteq F \) such that \( |F'| \leq K \) and \( \bigcup_{Fᵢ \in F'} Fᵢ = Y \) [9]. The reduction is as follows. Let each \( Fᵢ \) in F map to a role \( rᵢ \) and each element \( yⱼ \) in \( Y \) map to a permission \( pⱼ \). Let \( Y = \mathbb{P} \) and construct an SMER constraint \( c₁ = \{ r₁, \ldots, rₙ, K+1 \} \). Define the RBAC state \( φ = \langle R, PA, RH, C \rangle \) as: \( R = \{ r₁, \ldots, rₙ \} \) and \( (rᵢ, pⱼ) \in PA \) if \( yⱼ \in Fᵢ \) and \( RH = \emptyset \), and \( C = \{ c₁ \} \). Then compatibleᵦ(T) holds if \( F \) contains a cover for \( Y \) of size \( K \) or less. For example, T₂ is incompatible, whereas T₁ in Example 9 is compatible. Incompatibility may arise for several reasons. There is a possibility that for a given permission \( p \in \mathbb{P} \), the current RBAC state \( φ \) has no role \( r \) such that \( p \in PermSetᵦ(r) \) and PermSetᵦ(r) ⊆ \( \mathbb{P} \). p₁₁ is such a permission in the case of T₂.

In addition, it could happen that \( \{ p₁₁, p₂ \} \in \mathbb{P} \), RoleSetᵦ(p₁₁) = \{ r₁ \}, RoleSetᵦ(p₂) = \{ r₂ \}, and \( (\{ r₁, r₂ \}, 2) \in C \). In this case, only one of \( \{ p₁₁, p₂ \} \), but not both, can be shared.

**A. Adaptive Compatibility**

When incompatibility arises, administrators may resort to adjusting RBAC states to accommodate compatibility.

**Definition 15 (Adaptive Compatibility):** Given an RBAC state \( φ \), an interoperation request \( T \) and a set AR of administrators, we say \( T \) is adaptively compatible with \( φ \), if there exists a sequence of operations \( o₁o₂\ldots oₘ \), where \( oᵢ \in OP(AR) (1 \leq i \leq m) \), such that \( φ → AR φ' \) and compatibleᵦ(T). We denote this as compatibleᵦ(AR, φ, T). And we omit the label AR if it is clear from the context.

Given an incompatible request \( T \) in \( φ \), suppose that RMᵦ(T) = (rᴬ, Q). There definitely exists a set of administrators that makes compatibleᵦ(AR, φ, T) hold. A trivial approach is to let the whole set AR of administrators add a new role for each permission \( p \in \mathbb{P} \setminus PermSetᵦ(Q) \) and assign \( p \) to it. Take T₂ with the RBAC state in Figure 1 as background for instance. T₂ could be made adaptively compatible by \( arᵦ \)'s operations as follows: adds a role r₁ᵊ junior to r₁; assigns r₁ᵊ the permission p₁₁. Then \( RMᵦ(T₂) = \{ r₁ᵊ, r₁ \} \). Possibly this approach does not come up to our expectations. We leave the study on strategies for RBAC state evolution as future work.

However, due to the decentralized administration [6], [19], the available administrators are generally designated. With limited capabilities, an administrator can only perform restricted modifications of RBAC policies. Therefore, one natural problem is to decide whether, given a set AR of administrators, compatibleᵦ(AR, φ, T) holds. Nonetheless, this problem is computationally complex in general.

**Theorem 16:** Given an RBAC state \( φ \), a set AR of administrators and an incompatible request \( T \), determining compatibleᵦ(AR, φ, T) is NP-complete.

**Proof:** Since one has to verify compatibleᵦ(φ, T) according to Definition 15, it follows from Theorem 14 that the problem is NP-complete.

**Proposition 17:** Given an RBAC state \( φ \), a set AR of administrators, an incompatible request \( T \), and a fixed role mapping instance RMᵦ(T) = (rᴬ, Q), compatibleᵦ(AR, φ, T) holds if and only if \( ∀ p \in \mathbb{P} \setminus PermSetᵦ(Q) \) and \( RoleSetᵦ(p) \cap S(AR) \neq \emptyset \).

**B. Minimal Set of Administrators**

Given an incompatible request and a fixed RMᵦ(T), when B opts to pursue adaptive compatibility, we need to find a set AR of administrators who are capable of implementing changes to RBAC states. Obviously, the most senior administrator, such as \( ar₀ \) in Figure 1, is competent for all requests. Nevertheless, often the available administrators are limited to a set ARᵦᵦ. Many optimality criteria could apply to choosing suitable administrators. In this paper, we focus on saving administrative cost, which is measured by the number of administrators involved. That is, our problem is to find the minimal subset of ARᵦᵦ that is able to accomplish the administration.

**Theorem 18:** Given an RBAC state \( φ \), a fixed RMᵦ(T) and a set ARᵦᵦ of available administrators such that...
compatible_{\phi \rightarrow \phi'}^{AR}(T)$, finding a minimal subset $AR$ of $AR_{\forall \phi}$ such that $\text{compatible}_{\phi \rightarrow \phi'}^{AR}(T)$ is NP-complete.

Proof: Given a subset $AR$ of $AR_{\forall \phi}$, one can determine whether $\text{compatible}_{\phi \rightarrow \phi'}^{AR}(T)$ holds in polynomial time based on Proposition 17. Therefore the problem belongs to NP.

We now show that the problem is NP-complete by reducing minimal set cover (MSC) to one of its restricted case.

We first state the restrictions on our problem. Recall that $S(AR) = \{ r \mid C(AR) \cup \{ r \} \subseteq C(AR') \}$. Hence, we have $\bigcup_{\phi \in \phi} S(\phi) \subseteq S(AR)$. Then the first limitation is that $\bigcup_{\phi \in \phi} S(\phi) = S(AR)$. The second one is that, for every permission $p \in \mathbb{P} - \text{PermSet}_{\phi}(Q)$, $p$ is assigned to only one role in $AR_{\forall \phi}$'s administrative scope, i.e. $|\text{RoleSet}_{\phi}(p) \cap S(AR_{\forall \phi})| = 1$.

The reduction is as follows. Given $F$ and $Y$, construct the set $\mathbb{P} - \text{PermSet}_{\phi}(Q)$ and $AR_{\forall \phi}$ in the following way:

1. for each element $y_i \in Y$, create a role $r_i$ for it and add a new permission $p$ to $\mathbb{P} - \text{PermSet}_{\phi}(Q)$ with $\text{RoleSet}_{\phi}(p) = \{ r_i \}$,
2. create an administrator $ar_j$ for each element $F$ with the administrative scope $S(ar_j)$ corresponding to $F_j$.

The reduction can be accomplished in polynomial time. The administrator set $AR$ now corresponds to $F'$. Hence, if we can find $AR$ in polynomial time, finding $F'$ also takes polynomial time.

The general case of finding a minimal set of administrators is not equivalent to the MSC problem. The differences consist in the two restrictions mentioned above in the proof of Theorem 18. First of all, since $\bigcup_{\phi \in \phi} S(\phi) \subseteq S(AR)$, the minimal number in general case may be less than the solution to the restricted one, which is equivalent to MSC. For example, suppose that there is a request $T_3 = \{ p_2, p_3, p_4 \}$ and that $AR_{\forall \phi}$ for $T_3$ in Figure 1 is $\{ ar_2, ar_3, ar_4, ar_7 \}$. Then the minimal set is $\{ ar_2, ar_3 \}$, but the MSC problem corresponding to the restricted case is $\{ ar_2, ar_3, ar_7 \}$. Secondly, since $p \in \mathbb{P} - \text{PermSet}_{\phi}(Q)$, it is likely that $|\text{RoleSet}_{\phi}(p) \cap S(AR_{\forall \phi})| > 1$. This implies that we need not to cover the set $\text{RoleSet}_{\phi}(\mathbb{P} - \text{PermSet}_{\phi}(Q))$, as in the restricted case.

Still, we could apply the theoretic results about MSC [17] to our problem. As for the second distinction, we make the following observation. The MSC problem is often formulated as follows. Let $A = (a_{ij})_{m \times n}$ be a zero-one matrix:

$$a_{ij} = \begin{cases} 1 & \text{if } y_i \in F_j, \\ 0 & \text{otherwise}. \end{cases}$$

Then the MSC problem is to find a zero-one $n$-dimensional column-vector $x = (x_1, \ldots, x_n)^T$ such that $Ax \geq e$ and $\sum_{j=1}^n x_j$ is minimized, where $e$ is an all-one $m$-dimensional column-vector. And the solution is $F' = \{ F_i \mid x_i = 1 \}$. Now, we construct the matrix $A' = (a'_{ij})_{m \times n}$ in the following way instead so that our problem can be treated as an MSC problem:

$$a'_{ij} = \begin{cases} 1 & \text{if } \text{RoleSet}_{\phi}(p_i) \cap S(ar_j) \neq \emptyset, \\ 0 & \text{otherwise}. \end{cases}$$

Hence, the solution to the MSC problem $A'x \geq e$ with $\sum_{j=1}^n x_j$ minimized is also a solution to our problem. Suppose we obtain the solution $AR'$. Due to the first distinction (i.e. $\bigcup_{\phi \in \phi} S(\phi) \subseteq S(AR)$), $AR'$ is not necessarily the optimal solution. We can further optimize it: remove $ar'$ from $AR'$ if $S(AR' - \{ ar' \}) = S(AR')$.

Example 19: Continuing the example in Figure 1. Suppose the RBAC state $\phi$ updates to $\phi_1$ where the role-permission assignments $(r_5, p_{56})$, $(r_6, p_{56})$, $(r_2, p_{23})$ and $(r_3, p_{23})$ have been added, as shown in Figure 2. Now a request $T_4 = \{ (p_{23}, p_{56}, p_2, p_1) \}$ arrives. It follows that $RM_{\phi_1}(T_4) = (r_4, \emptyset)$. Assume that some administrators want to adjust the state. $\mathbb{P}_{56} - \text{PermSet}_{\phi_1}(\emptyset) = \{ p_{56}, p_{56}, p_1, p_1 \}$. Suppose the set $AR_{\forall \phi}$ of available administrators is $\{ ar_2, ar_3, ar_5, ar_6, ar_7 \}$. Then the formulation $A'x \geq e$ of this problem is as follows:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \ 0 & 0 & 1 & 1 & 0 \ 1 & 0 & 0 & 0 & 0 \ 0 & 0 & 0 & 1 & 0 \ 0 & 0 & 0 & 0 & 1 \ \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

Suppose the solution is $x = (1, 1, 1, 0, 0)^T$, which implies that $AR = \{ ar_2, ar_3, ar_5, ar_7 \}$. And since $S(AR - \{ ar_2 \}) \subseteq S(AR)$, the minimal subset of administrators is actually $\{ ar_2, ar_3, ar_5 \}$.

Figure 2. The RBAC state $\phi_1$ deriving from $\phi$ in Figure 1.

C. A Discussion on Administrative Operations for Adaptive Compatibility

There are three self-evident requirements for administrative operations:

1. $R \subseteq R'$,
2. $\forall r \in R \ [\text{PermSet}_{\phi}(r) = \text{PermSet}_{\phi'}(r)]$, and
3. $C' \Rightarrow C^2$.

Even though the requirements rule out some improper operations, there still exist many choices of operations. Given an instance $(r_A, Q)$, $AR$ can create separate roles for permissions in $\mathbb{P} - \text{PermSet}_{\phi}(Q)$. Another option is to split one role in $\text{RoleSet}_{\phi}(p)$ so that one of its junior roles could be mapped afterwards. No doubt that both the two choices meet the requirements.

Example 20: Consider an RBAC state $\phi_2$, shown in Figure 3 (a), and a request $T_5 = \{ (p_6, p_{66}, p_{67}, p_{9}, p_{912}, p_{112}) \}$. A role mapping instance for $T_5$ is $(r_A, \{ r_9 \})$. Since $\text{PermSet}_{\phi_2}(r_9) = \{ p_6, p_{66}, p_{912}, p_{112} \}$, $T_5$ is incompatible in $\phi_2$ and changes to $\phi_2$ could be made. Figure 3 (b), (c) and (d) show three possible states following different sets of operations, respectively. While $\phi_2$ is an instance of the first method mentioned above, $\phi_{2e}$ and $\phi_{2d}$ illustrate the second solution. The

$C^2 \Rightarrow C$ means that an RBAC state satisfies a set $C'$ of SMER constraints only if it also satisfies $C$ [3, 15].
administrator \( ar_1 \) constructs \( \phi_{2b}' \) by the following operations: creates the role \( r_{n1b} \), and assigns \( p_{11}' \) and \( p_{12}' \) to it. And \( ar_6 \) makes \( \phi_{2b}' \) by creating a role \( r_{n12} \) junior to \( r_6 \) and assigning it \( p_{11}' \), \( p_{21}' \) and \( p_{22}' \). \( \phi_{2b}' \) is derived similarly but with \( p_{11} \) assigned to \( r_{n2d} \).

\[
\begin{align*}
\text{(a: } \phi_2) & : p_{11}', p_{21}', p_{31}' & \quad \text{(b: } \phi_{2b}) & : p_{11}', p_{21}', p_{32}' \\
\text{(c: } \phi_{2b}') & : p_{11}', p_{21}', p_{32}' & \quad \text{(d: } \phi_{2b}'') & : p_{11}', p_{21}', p_{32}'
\end{align*}
\]

Figure 3. An RBAC state \( \phi_2 \) that derives from \( \phi \) in Figure 1 by adding the role-permission assignment \( \{ (r_{n1}, p_{11}'), (r_{n1}, p_{12}') \} \). Note that \( r_6 \) possesses \( \{ p_{11}' \}, p_{21}' \) because \( r_6 \not\simeq r_{11} \).

However, the second solution is preferred than the first one for two reasons. Firstly, though simple enough, the first solution would add meaningless roles (eg. \( r_{n1b} \) in \( \phi_{2b}' \)) and build additional separate role hierarchies with no semantics (eg. \( \{ r_{n1b} \} \)). Secondly, more senior administrators are needed to manage new roles in the first solution. In the second scenario, new roles are within the administrative scopes of the split roles’ administrators. For example, \( r_{n1d} \) and \( r_{n2d} \) in the state \( \phi_{2d}' \) are within \( ar_6 \)'s administrative scope, whereas only \( ar_1 \) (and more senior administrators) could control the role \( r_{n1b} \) in the state \( \phi_{2b}' \).

VI. RELATED WORK

Joshi et al. [12] studied an XML-based RBAC language for access control in multi-domain environments where each domain employs RBAC policies. Piromruen and Joshi [18] proposed a time-based policy mapping framework between two domains where Generalized Temporal Role Based Access Control (GTRBAC) policies have been employed. Du and Joshi [7] studied the problem of mapping a request for a set of permissions to a minimal set of roles in presence of hybrid hierarchies. But the administration and SMER constraints (and thus SoD constraints) were not considered there.

Generally speaking, proposed role mapping generation approaches in literature can be categorized into two classes. The first one depends on security administrators who manually select role mappings [13], [22], [23], and the other automatically maps roles based on certain assumptions and principles [21]. While manual selection is simple and easy to implement, the second approach enables convenient generation of role mappings.

Shafiq et al. [21] proposed a centralized RBAC-based framework for building up secure interoperability. This framework is capable of composing a secure interoperation policy from RBAC policies of multiple domains. Also, they analyzed trade-offs between loss of autonomy and the degree of interoperation, and proposed a set of formal parameters for the description of autonomy. Hu et al. [10] proposed a method for setting up interoperating relationships between domains while preserving domains’ security with respect to separation of duty constraints. Nevertheless, no analysis of the role mapping generation from the computational perspective was given.

Shehab et al. [22], [23] presented a distributed framework for secure interoperation. The framework enables domains to make localized access control decisions based on the user’s access history. However, little attention has been paid to the setup of inter-domain role mappings. We have a bias on the computational analysis. One difference is that this paper works on the set of roles that can be discovered to be mapped. Hence, this work can be a complement of Shehab et al’s works.

Later, Shehab proposed some role mapping discovery protocols and their implementation in the context of web services [24], [25]. These works focus on the implementation of role mapping mechanisms for secure interoperation. Thus they corroborated the importance and practicableness of role mappings. Other works that highlight the importance of inter-domain role mappings include [11], [16], [14]. Jin and Ahn [11] presented a role-based access management framework for secure digital information sharing in collaborative environments. The framework depends on mappings between collaborator roles and normative collaboration roles. Pan et al. [16] proposed semantic access control for interoperation based on RBAC and employed a role mapping table. Based on role mappings, Li et al. [14] proposed a framework for secure collaboration in virtual organizations.

VII. CONCLUSION

Mapping roles is a basic approach to secure interoperation in RBAC-based collaborating environments. In this paper, we studied how to generate role mapping instances, mostly from computational perspective. We showed this problem can be computationally complex. We also considered the administrative issues when \( B \) is willing to adjust its RBAC state to accommodate for interoperation. The result implies that it is hard to determine whether \( B \) can enable maximal interoperability through modifications of its RBAC state, and that to save administrative cost is not easy either.

Several on-going future work may be interesting. First of all, it is worthwhile to design heuristic algorithms for the role mapping generation. Second, through various means of RBAC state modifications exist, there is no formal measurement to evaluate them; and more importantly, one needs to take into account how to preserve the semantics of RBAC states and confine the administrative burden to an acceptable level.

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